Hybrid Marketplaces with Free Entry of Sellers

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Abstract

We study a hybrid marketplace such as Amazon selling its own products and setting commissions on sellers engaged in monopolistic competition with free entry. For a large class of microfoundations based on a representative agent, the introduction of products by the marketplace is neutral on consumer welfare for a given commission, but exerts an ambiguous impact through its changes: a "demand substitution mechanism" pushes for a higher commission, but an "extensive margin mechanism" pushes for a lower commission aimed at attracting new sellers and more purchases on the marketplace. With constant demand elasticities, a hybrid marketplace sets a lower (higher) commission rate and increases (decreases) consumer welfare compared to a pure marketplace if its products face a less (more) elastic demand.

Key words: Hybrid marketplaces, 3P Sellers, Commissions, Entry, Monopolistic Competition.

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1 Introduction

A hybrid marketplace is monetized through percentage commissions on third party sales and through direct sales of its own products and services. In the case of Amazon, as well as other prominent platforms (as those by Apple and Google), this double role as "umpire and player" has been at the center of a lively debate. The common presumption is that a hybrid marketplace would systematically promote its own products or increase commissions on third party ones to favor its own sales, and this may harm consumers in the long run (according to the New Brandesian view of Khan, 2016). In this work we ask whether this is consistent with the endogenous market structure emerging on a hybrid marketplace open to third party sellers.[2] Contrary to the common presumption, we show that the introduction of own products can actually increase both consumer welfare and total welfare through a reduction of commissions on sellers which reduces all prices and expands gains from variety.[3]

Recent important works by Anderson and Bedre-Defolie (AB, 2021) and Zennyo (2021) have introduced tractable frameworks based on a Logit model of product differentiation to address these issues. In both these works, a hybrid marketplace acts as a Stackelberg leader in selecting commissions and prices, and faces endogenous entry of third party sellers engaged in imperfect competition. Zennyo (2021) adopts a commission on units sold and shows that a hybrid marketplace is neutral on commissions and consumer welfare, while AB (2021) adopt a percentage commission on revenues (the empirically relevant case) and argue that a hybrid marketplace sets excessive commission rates to shift demand toward its own products, which reduces consumer welfare. We unveil the nature of this apparent contradiction by developing a microfoundation of demand systems which nests the Logit case and an entire class of alternative ones, and we derive conditions under which a hybrid marketplace can either increase or decrease welfare compared to a pure marketplace.

More formally, we adopt a representative agent framework based on a quasi-linear indirect utility depending on additive aggregators of the prices of all the products sold on the marketplace. The sellers are engaged in monopolistic competition with free entry.[4] For a given commission rate, the entry and pricing

[2] The sale of "house brands" alongside the sale of third party brands has a long history in retailing (for supermarkets, drug stores, hardware stores, department stores, and more), and this vertical integration is usually considered a pro-competitive feature. The concern around "house brands" offered by Amazon appears to be related with its market power in online distribution and its ability to affect entry and investment of third party sellers.

[3] We should remark that part of the social and political concern about dominant marketplaces such as Amazon or Google is about other ways in which they can disadvantage third party sellers, through favorable positions on the first screen or biased recommendations and through learning the details of the characteristics of the customers of sellers and then using that information to develop better versions of their own products. We will not directly deal with these issues in the present analysis.

[4] Representative agent models of monopolistic competition with indirect additivity were introduced in Bertoletti and Etro (2017). It should be emphasized that AB (2021) and Zennyo (2021) rely on discrete choice models with random utility augmented respectively with search costs for consumers and a consideration set depending on consumers' search efforts. For

strategies of the marketplace are neutral on consumer welfare, a consequence of a result applying to aggregative games with free entry (Etro, 2008; Anderson *et al.*, 2020). The only impact of the introduction of products by the marketplace on consumer welfare occurs through a change in the commission set on sellers. When this is increased, consumers are harmed through higher prices and lower gains from variety, while a reduction of the commission reduces prices and expands the gains from variety, increasing consumer welfare as well as total welfare. This result is powerful because independent from details on the demand conditions (the welfare impact depends on an observable feature as the change in commissions). Moreover, we show that either outcome is possible because the introduction of products by the marketplace exerts two effects. On one side, there is an incentive for the marketplace to increase the commission and shift demand toward its own products, but, on the other side, there is an incentive to reduce the commission to attract new sellers collecting more commission revenues on the extensive margin and to expand purchases by customers on the marketplace. One can regard the former as a *demand substitution effect* which harms consumers biasing their purchases toward the marketplace's products, and the latter as an *extensive margin effect* which benefits consumers expanding purchases of all products.

Under additional restrictions on the microfoundation we can obtain more precise results on the conditions under which each effect is dominant. In the prominent case of demands with a constant elasticity, if the marketplace faces the same elasticity as the sellers, its products are introduced at the same price and the commission rate is left unchanged. However, when the marketplace faces a less elastic demand than the sellers (for instance due to a reputational advantage of Amazon on its platform), the extensive margin effect is dominant and the marketplace sets higher markups but reduces the commission on third party sales to attract more purchases. Instead, under a Logit demand system the demand substitution effect is dominant and the commission is increased, which is consistent with findings by AB (2021). We explore extensions to specific commissions, where the neutrality applies in the Logit case (which is consistent with findings by Zennyo, 2021), strategic interactions between sellers, alternative timing, product selection by the marketplace and advertising for product discovery.

Our findings suggest that the presumption that a hybrid marketplace tends to favour its own products through worse conditions or higher commissions for third party sellers lacks a solid foundation. For instance Amazon sets different commission rates across wide product categories, and these rates have been quite stable over time and, more important for the implications of our model, not correlated with the introduction of products by Amazon. Public data from Amazon in the US reveal that the commission rates between 2017 and 2021 have been constant at 8% for consumer electronics, cameras, cell phone devices and video game consoles, 12% for industrial & scientific products, 15% for

empirical applications of related models on Amazon see Lee and Musolff (2021) and Gutierrez (2021).

books, home & garden, office products, music, sports, toys and much more, and 45% for Amazon device accessories. Changes in commissions during the last five years have been reductions (below a price threshold) from 15% to 8% for baby products, beauty and health & personal care, from 20% to 15% for sports collectibles, and (above a price threshold) from 15% to 10% for the category furniture & decor and from 20% to 5% for jewelry, with the only increase being from 15% to 17% for clothes and from 6% to 8% for personal computers.[5] Remarkably, private label products had been introduced for product categories with unchanged commissions, as well as for product categories with a reduction of the commissions and for clothes, but not for personal computers. Such a state of affairs, broadly confirmed in other countries, suggests that there is not a significant correlation between changes in commission rates and market shares of Amazon by product category.

The work is organized as follows. Section 2 reviews the related literature. Section 3 presents the structure of the model. Section 4 derives the key results. Section 5 discusses extensions. Section 6 concludes.

2 Literature review

This work is related to the literature on platforms with competing sellers (Hagiu, 2009; Belleflamme and Toulemonde, 2016; Belleflamme and Peitz, 2019; Teh, 2020; Bisceglia *et al.*, 2021; Jeon and Rey, 2021) and especially to the expanding literature on online marketplaces (AB, 2021, 2022; Zennyo, 2021; Kittaka and Sato, 2021; Hervas-Drane and Shelegia, 2022; Lam and Liu, 2021; Tremblay, 2021; Kang and Muir, 2021; Madsen and Vellodi, 2021; Ronayne and Taylor, 2021; Hagiu *et al.*, 2021; Peitz and Sobolev, 2022). A common theme emerging in this literature, and confirmed in the present work, is that the business model of an online marketplace, based on monetization of all products on the platform including those of third party sellers through commissions, is a key factor that disciplines the incentives to introduce, price and promote its own products.[6]

In a static perspective, it has been emphasized that entry by the marketplace tends to materialize in case of cost efficiencies or demand advantages that benefit also consumers (Hagiu and Wright, 2015; Etro, 2021a; Hervas-Drane and Shelegia, 2022; AB, 2022). In a dynamic perspective, it has been emphasized that even when imitative entry by the marketplace disincentivizes investment by sellers, there is an incentive to commit to a limited copycat activity internalizing the impact on future product creation for the same marketplace, which

[5] There was also a momentarily increase from 15% to 18% for the commission on shoes, handbags & sunglasses in 2018, but the commission was decreased to its initial level in 2020. Note that while referral fees have remained mostly constant across time, other fees, as those for Fulfilment By Amazon, have been increasing. However, those fees are not tailored by product category, so they are not relevant for our arguments.

[6] Recent evidence that Amazon better internalizes the interest of consumers in setting prices of its own products compared to third party sellers is in Cabral and Xu (2021), who study prices of face masks and hand sanitizers at the beginning of the pandemic phase. For an early analysis of how business models affect the incentives of digital platforms see Caffarra (2019).

creates benefits also for consumers (Jiang *et al.*, 2011; Etro, 2021a; Madsen and Vellodi, 2021).

In a more general perspective, Hagiu *et al.* (2021) have shown that hybrid marketplaces create gains for consumers through more competition on the platform, but could raise concerns related to self-preferencing and, in the absence of commitment policies, excessive imitation of sellers. While their framework is centred on search by consumers across products by sellers with market power and competitive fringes of rivals active also through a direct channel, our framework is centred on free entry of monopolistically competitive sellers providing differentiated goods only on the marketplace. A common conclusion is that a hybrid marketplace such as Amazon can create benefits for consumers, and potential concerns should be addressed by antitrust policy through behavioral remedies (and not structural ones).

Our results resonate well with those of Shopova (2021) and Hervas-Drane and Shelegia (2022). Shopova (2021) explores a vertical differentiation model and shows that a marketplace has an incentive to introduce low quality private labels and reduce commissions on vertically differentiated sellers, generating an increase in consumer welfare: the marketplace introduces an additional variety and reduces commissions because it internalizes the lower demand of the sellers and the higher pass-through on their prices. Hervas-Drane and Shelegia (2021) confirm in a different framework that a hybrid marketplace affects the trade-off in setting commission rates and may reduce them to recover entry of sellers.

A broader application of our findings is about vertical integration. After all, an online marketplace can be regarded as an upstream monopolist that provides downstream distribution services as an input for differentiated downstream producers, and the provision of its own products, either new ones or absorbing products by others, amounts to a form of vertical integration. Our analysis suggests that a vertical merger with one of the producers may either raise or reduce the price of the input (here the commission) under endogenous entry. A *raising rivals' cost effect* pushes for a higher price of the input to divert demand from the other downstream firms (and for a given number of products, the impact on the price of the monopolist would depend on the balance of the elimination of double marginalization and increased demand for the product). However, when entry of downstream producers is endogenous, the change in the price of the input affects entry or exit of other producers, and we obtain a new *extensive margin effect* which pushes for a lower price of the input in the attempt of attracting more producers and expand revenues on them. The welfare implications of the merger depend on which effect dominates.[7] A recent work by Kang and Muir (2021) has explored the impact of an analogous vertical integration when a platform sets non-linear fees on homogenous downstream producers with

[7] This effect is absent in the literature on vertical integration with an exogenous number of firms. I am grateful to Michele Polo and Lawrence J. White for pointing out this application to vertical integration. On the recent literature on vertical mergers see the September 2021 Special Issue of the *Review of Industrial Organization* on the U.S. Vertical Merger Guidelines, and in particular the works by Salinger (2021), Shapiro (2021) and Moresi and Salop (2021) on the trade-off between raising rivals' cost and elimination of double marginalization effects.

private information on costs: in that case the integration benefit consumers by avoiding a form of double marginalization and reducing final prices.

Our welfare analysis is also related to the theoretical literature on market competition with free entry (see Spence, 1976, Dixit and Stiglitz, 1977, Bertoletti and Etro, 2016, 2017) and Stackelberg leadership in aggregative games with free entry (Etro, 2008; Tesoriere, 2008; Ino and Matsumura, 2012; Anderson *et al.*, 2020; Alfaro, 2020; Alfaro and Lander, 2021), and to the related empirical literature (see Berry and Waldfogel, 1999, and Dutta, 2011). In particular, Lee and Musolff (2021) have recently provided an empirical analysis of self-preferencing by Amazon in a nested Logit framework with free entry of heterogeneous sellers, and their results suggest that practices adopted by Amazon have not harmed consumers, even without accounting for endogenous commissions by the marketplace.[8] More generally, our work provides a framework that can be used to explore how policy commitments affect sellers active in markets with free entry and what is their impact on consumer welfare, an issue emerging in various fields, including industrial and trade policy.

3 The Model

Let us consider a hybrid marketplace offering $n > 0$ products, of which $m \in [0, n)$ directly provided by the same marketplace and the remaining ones provided by third party sellers engaged in monopolistic competition.[9] As in standard partial equilibrium models *à la* Spence (1976) we adopt quasilinear preferences for a representative customer of the marketplace. We express preferences through an indirect utility that is a convex function of the price vector $\mathbf{p}$ of all products sold on the marketplace:

$$V = G\left(D(\mathbf{p})\right) + E \tag{1}$$

where $D(\mathbf{p})$ is a price aggregator, $G(D)$ is an increasing and concave transformation and E is expenditure, assumed large enough to allow purchases of an outside *numeraire* good. The aggregator is assumed indirectly additive (IA) in

[8] Gutierrez (2021) has provided an empirical analysis of vertical integration by Amazon (in the spirit of the work by Crawford *et al.*, 2018) in a nested Logit framework with endogenous commissions by the marketplace, and his results suggest that consumer welfare is lower in a pure marketplace compared to a hybrid one, even without accounting for endogenous entry of sellers. See also Crawford *et al.* (2022) for a major work on Amazon entry and its effects on sellers and consumers.

[9] Amazon intermediates about 44% of e-commerce sales in the U.S., and the majority of these sales are by third party sellers hosted on the platform, with 36% sales by Amazon as a first party retailer and 5% sales through private labels by Amazon in 2020 (Gutierrez, 2021). Since we allow for heterogeneous costs and demand of the products of the marketplace, we can interpret these either as private label products or products retailed by the marketplace.

the prices of the products as in:[10]

$$D(\mathbf{p}) \equiv \sum_{j=1}^{n} v_j(p_j) + H \tag{2}$$

which depends on the sum of incremental surplus functions of all products and a constant $H > 0$ reflecting an exogenous surplus obtained from the marketplace or from other goods purchased by consumers outside the marketplace.[11] For tractability we assume that all third party products are symmetric with a common surplus function $v(p_j)$ which is positive, decreasing and convex in the price p_j, but we allow for heterogeneous products by the marketplace with surplus functions $v_j(\bar{p}_j)$ for $j = 1, 2, .., m$ that are also assumed positive, decreasing and convex in the prices $\bar{p}_j$ set by the marketplace.

Applying the Roy's identity to these quasilinear IA preferences, each product i faces the direct demand:

$$q_i(p_i) = |v_i'(p_i)| \, G'(D(\mathbf{p})) \tag{3}$$

which emphasizes that the additive aggregator crucially determines both welfare and the demand system.

We will illustrate this microfoundation with a logarithmic transformation that will be widely employed later on:

$$G(D) = \log D \tag{4}$$

This provides the loglinear preferences used by Nocke and Schutz (2018) to study multiproduct pricing with imperfect substitutability. They deliver the demand functions:

$$q_i(p_i) = \frac{|v_i'(p_i)|}{\sum_j v_j(p_j) + H} \tag{5}$$

which are clearly decreasing in the aggregator. In the particular specification with exponential functions $v_j(p) = e^{-p/\mu}$ where $\mu > 0$ determines product differentiation, the demand function becomes $q_i(p_i) = e^{-p_i/\mu}/\mu(\sum_j e^{-p_j/\mu} + H)$, and the model is isomorphic to one based on a Logit foundation (Zennyo, 2021; AB, 2021). Of course, other functions would deliver other relevant demand systems, determining the perceived demand elasticities of each product.[12]

[10] We follow Nocke and Schutz (2018) in assuming quasi-linearity for partial equilibrium analysis. However, we could allow for income effects across consumers without affecting the main results, following the analysis of monopolistic competition under indirect additivity and outside goods introduced in Bertoletti and Etro (2017).

[11] We could obtain analogous results adopting a quasilinear direct utility that is a function of an aggregator of quantities as in Spence (1976). But notice that the underlying preferences and demand systems are not overlapping (unless the monotonic transformation is linear).

[12] On the correspondence between discrete choice models and representative agent models based on indirectly additive aggregators see Thisse and Ushchev (2016).

3.1 Surplus functions

In the monopolistic competition framework adopted here, sellers facing the general demand function (3) set prices taking as given the price aggregator, therefore what matters for pricing is the perceived demand elasticity, that is the elasticity of the $v'(p)$ function. The underlying function $v(p)$ determines the additional surplus obtained by consumers in function of the price, therefore its shape determines both the surplus elasticity $\zeta(p) \equiv -\frac{v'(p)p}{v(p)}$ and the elasticity of demand $\varepsilon(p) \equiv -\frac{v''(p)p}{v'(p)}$, both of which are positive under our assumptions.

A classic specification is based on power functions:

$$v(p) = p^{1-\varepsilon} \tag{6}$$

with $\varepsilon > 1$, and delivers an isoelastic demand function, with a demand elasticity $\varepsilon(p) = \varepsilon$ and a surplus elasticity $\zeta(p) = \varepsilon - 1$. In case of exponential functions:

$$v(p) = e^{-p/\mu} \tag{7}$$

with $\mu > 0$ parametrizing product differentiation, the two elasticities are $\varepsilon(p) = \zeta(p) = p/\mu$ and increase in the price. Another useful case is based on the translated power surplus:

$$v(p) = \frac{(a-p)^{1+\gamma}}{1+\gamma} \tag{8}$$

where $a > 0$ and $\gamma > 0$ parameterize willingness to pay and shape of demand, with elasticities such that $\varepsilon(p)/\zeta(p) = \gamma/(1+\gamma)$. We will repeatedly use these specifications for illustrative purposes.

It is easy to compute that the surplus elasticity changes with the price according to $\zeta'(p) = \frac{\zeta(p)}{p}\left[1 + \zeta(p) - \varepsilon(p)\right]$, and therefore it can be either constant in the price (under power functions) or variable (as in the other examples above) depending on its relation with the demand elasticity, which can also be either constant or variable in the prices.

3.2 Technology and timing

Each seller bears a fixed cost of entry $f > 0$.[13] The marketplace provides a good j at marginal cost $\bar{c}_j \geqslant 0$ setting the price $\bar{p}_j$, while any third party seller i provides its good at a common marginal cost $c \geqslant 0$ setting the price p_i under monopolistic competition. The revenues of the sellers are subject to a uniform percentage commission at rate $\tau \in [0, 1]$ paid to the marketplace. We focus on monopolistic competition with the understanding that there is a large number of sellers whose pricing has negligible effects on the price aggregator, but we will

[13] With a population of customers, it would be the ratio of fixed costs and population that would matter for entry. While the cost of entry on a marketplace may be low, this should also reflect the opportunity cost of not using other solutions. Notice that we could allow for heterogeneity in costs as well as demand of some sellers as long as a zero profit condition binds on a competitive fringe of small sellers.

later extend the analysis to the case of Bertrand competition between a limited number of sellers (which makes it useful to use a discrete number of firms in the baseline model).

The timing of the benchmark game is the following: 1) the marketplace sets the uniform commission rate on third party sellers; 2) the marketplace sets the prices of its own products; 3) entry of sellers takes place and 4) the sellers set their prices under monopolistic competition. This reflects the rather stable commitment of Amazon to its commission rates per product category, and its ability to introduce own products affecting entry and pricing of sellers. We will later extend the model with price decisions of the marketplace taking place after entry of sellers, which will strengthen the incentives to reduce commissions. We will also consider a preliminary stage of product selection by the marketplace, when also the latter bears fixed costs of product introduction.

4 Equilibrium analysis

In this section we solve for the subgame perfect equilibrium of the benchmark model by backward induction. Our final aim is to compare the choices made by a hybrid marketplace offering multiple own products and a pure marketplace offering only products by third party sellers.

4.1 Pricing and entry of the sellers

Given the strategies of the marketplace and the number of sellers, each seller i sets the price p_i to maximize profits:

$$\pi(p_i) = [(1-\tau)p_i - c]\,|v'(p_i)|\,G'(D(\mathbf{p})) - f \tag{9}$$

taking as given the price aggregator (2) under monopolistic competition. This provides a common price rule $p = p(\tau)$ for each product satisfying:

$$p = \frac{\varepsilon(p)c}{(\varepsilon(p)-1)(1-\tau)} \tag{10}$$

where the demand elasticity $\varepsilon(p)$ is now assumed larger than unity in equilibrium with a positive marginal cost (but approaching unity for zero marginal cost). The independence of pricing from the prices and number of the other products relies on the IA property of the price aggregator and the assumption of monopolistic competition. The positive impact of the commission on the price depends on the shape of the demand function, and can be computed as $p'(\tau) = \eta(p(\tau))p(\tau)/(1-\tau)$, where the pass-through elasticity of the price with respect to the marginal cost $\eta(p) \equiv \partial \ln p/\partial \ln c$ can be easily shown to be less (more) than unitary if $\varepsilon'(p)$ is positive (negative) as long as the marginal cost is positive.

For instance, under a power surplus function (6), we obtain the price rule:

$$p(\tau) = \frac{\varepsilon c}{(\varepsilon-1)(1-\tau)}$$

and the cost pass-through is full with $\eta(p(\tau)) = 1$. Instead, with the exponential surplus function (7), the price of the sellers is:

$$p(\tau) = \mu + \frac{c}{1-\tau}$$

as in common Logit models, and the pass-through is incomplete with elasticity $\eta(p(\tau)) = 1/[1 + (1-\tau)\frac{\mu}{c}]$. Finally, in case of translated power functions the price is:

$$p(\tau) = \frac{a + \frac{\gamma c}{1-\tau}}{1+\gamma}$$

with incomplete pass-through and $\eta(p(\tau)) = 1/[1 + (1-\tau)\frac{a}{\gamma c}]$.

Using the price rule, we can express the profits of each seller as a decreasing function of the commission rate $\pi(p(\tau))$, and this expression decreases also in the value of the price aggregator due to the concavity of $G(D)$. Given the commission and the prices of the products of the marketplace, entry of new third party sellers increases n and, therefore, the value of the price aggregator:

$$D(\mathbf{p}) = \sum_{j=1}^{m} v_j(\bar{p}_j) + (n-m)v(p(\tau)) + H \tag{11}$$

reducing the gross profits of each seller until they match the fixed cost f (focusing, of course, on cases where entry takes place). Accordingly, free entry pins down the equilibrium value of the aggregator as a function of the commission rate $D = D(\tau)$ such that:

$$[(1-\tau)p(\tau) - c]\,|v'(p(\tau))|\,G'(D) = f \tag{12}$$

The equilibrium aggregator is decreasing in the commission with derivative:

$$D'(\tau) = \frac{-\varepsilon(p(\tau))D(\tau)}{(1-\tau)\sigma(D(\tau))} < 0 \tag{13}$$

where we introduced an index of curvature for the monotonic transformation $\sigma(D) \equiv \frac{-G''(D)D}{G'(D)} \geqslant 0$, which is actually constant and unitary under loglinear preferences.

An increase of the commission exerts a direct negative impact on the profits of the sellers (while the impact through prices is null by the envelope theorem), which reduces the value of the aggregator independently from the provision and the pricing of products by the marketplace. This implies that consumer welfare, which here is also "user welfare" due to zero profits of the sellers, amounts to $V = G(D(\tau)) + E$, independent from products and prices of the marketplace for a given commission. Given the generality of this result, we formalize it as follows:

Proposition 1. *Under monopolistic competition with free entry of sellers on a marketplace serving customers with quasilinear IA preferences, the introduction of products by the marketplace is neutral on consumer welfare for a given commission.*

To the extent that the marketplace is not changing commissions while introducing new products, there are no consequences on the prices of the sellers due to monopolistic competition and on the IA price aggregator due to free entry. Therefore, this framework, as already those of AB (2021) and Zennyo (2020), implies that the hybrid marketplace is neutral on consumer welfare, independently from the prices and the qualities of the products of the marketplace. The only impact of the introduction of new products is to crowd out some sellers and affect their number, which can be derived as follows:

$$n(\tau) - m = \frac{D(\tau) - \sum_{j=1}^{m} v_j(\bar{p}_j) - H}{v(p(\tau))} \tag{14}$$

and is assumed positive in equilibrium to focus on interesting cases.[14] The introduction of a product generating lower (higher) surplus than a seller expands (reduces) the total number of products because it opens more space for entry of third party sellers, but with no ultimate impact on the aggregator. The neutrality on the aggregator and consumer welfare relies on a well known property of this class of aggregative games with a Stackelberg leader and endogenous entry of followers - for related statements of the neutrality property see Etro (2008, 2011) and Anderson *et al.* (2020).[15]

Accordingly, in the rest of the work we will examine the indirect impact that a hybrid marketplace has on user welfare through changes of the commission rate. We should emphasize that the assumption of a competitive fringe of symmetric sellers implies zero profits of sellers and that consumer welfare corresponds to user welfare, that is the sum of the welfare of consumers and sellers. More realistically, we could introduce few large third party sellers that obtaining higher profits. As long as the zero profit condition remains binding on the fringe of small sellers, the main analysis goes through, but user welfare is now the sum of consumer welfare and the positive profits of the large sellers. In such a case, a reduction (increase) of the commission rate would increase (reduce) not only consumer welfare, but also the profits of the large third party sellers, and therefore user welfare. Accordingly, one can apply our results on consumer welfare to results on user welfare in a more general model.[16]

[14] Otherwise the marketplace becomes a pure retailer. Notice that a higher surplus from goods purchased outside the marketplace H reduces the total number of products sold on the marketplace because it reduces the effective demand of each product. Accordingly, we are implicitly assuming either that H or τ are not too high. As in standard monopolistic competition models, the selection of the entrants is irrelevant because all sellers are assumed symmetric.

[15] The neutrality of the aggregator applies not only under the so-called "independence from irrelevant alternatives", but with any demand systems based on a single symmetric aggregator. A more general microfoundation involves Gorman-Pollak preferences (see Bertoletti and Etro, 2021; Fally, 2022). The neutrality applies also under Bertrand competition between sellers discussed in Section 5. Related applications are, for instance, in Etro (2011), Ino and Matsumura (2012), Alfaro (2020) and Alfaro and Lander (2021).

[16] I am thankful to Yusuke Zennyo and anonymous referees for comments on this extension.

4.2 Pricing by the marketplace

Taking into account that third party products generate commission revenues and own products generate direct profits, we can rearrange the gross profits of the marketplace using (14) as follows:

$$\begin{aligned} \Pi &= \left[\tau(n(\tau)-m)p(\tau)\left|v'\left(p(\tau)\right)\right| + \sum_{j=1}^{m}(\bar{p}_j-\bar{c}_j)\left|v_j'(\bar{p}_j)\right|\right] G'(D(\tau)) \quad (15) \\ &= \tau\zeta(p(\tau))[D(\tau)-H]G'(D(\tau)) + I(\mathbf{p},\tau)G'(D(\tau)) \end{aligned}$$

where we remind the reader that $\zeta(p)$ is the elasticity of the surplus function. The first term in the second line of (15) represents the commission revenues of a pure marketplace, where $\tau\zeta(p(\tau))$ determines the impact of the commission on the relative revenues per seller and the aggregator $D(\tau)$ determines the impact of the commission on entry of sellers, with $D(\tau) > H$ under our assumptions. The second term represents the profits generated by the products of the marketplace net of the lost commission revenues, where:

$$I(\mathbf{p},\tau) \equiv \sum_{j=1}^{m}\left[(\bar{p}_j-\bar{c}_j)\left|v_j'(\bar{p}_j)\right| - \tau\zeta(p(\tau))v_j(\bar{p}_j)\right] \quad (16)$$

is an index of differential profits between own and third party products. Such an index is corrected to internalize the impact of the products supplied by the marketplace on the (reduced) entry of third party sellers and therefore on the (lost) commission revenues. In particular, setting a higher price generates lower surplus from the product of the marketplace, which attracts a larger number of sellers and more commission revenues. And, assuming that $\tau\zeta(p(\tau))$ increases in the commission (as the case in the relevant range), a higher commission reduces the index of differential profits.

The platform selects the prices of its own products to maximize profits (15) taking as given the price aggregator $D(\tau)$, since this is expected to be constant under free entry for a given commission, but taking into account the opportunity cost of losing commission revenues on sellers's products. Since only $I(\tau)$ is affected by the prices of the marketplace, its maximization for any product $j = 1, 2, ..., m$ provides rules $\bar{p}_j = \bar{p}_j(\tau)$ that, assuming an interior solution, satisfy:

$$\bar{p}_j = \frac{\varepsilon_j(\bar{p}_j)\bar{c}_j}{\varepsilon_j(\bar{p}_j) - 1 - \tau\zeta(p(\tau))} \quad (17)$$

where $\varepsilon_j(p) \equiv -\frac{v_j''(p)p}{v_j'(p)}$ is the demand elasticity for the product j of the marketplace. The fact that the marketplace avois double marginalization on its own products pushes for a low price, but the fact that it has a lower opportunity cost of increasing prices (since demand is partly shifted to sales monetized with commissions) pushes for a high price. The shapes of the demand and surplus elasticities drive the two effects determining which one is dominant.

An interesting case to focus on is the one of a good produced at the same cost and generating the same surplus whether supplied by the marketplace or by the seller (arguably the case where the marketplace acts as first party retailer). For such a good, marketplace and seller set the same price when the commission is null, but the comparison is ambiguous for a positive commission. In particular, the price of the marketplace is lower than the price of the seller if:

$$\bar{p} = \frac{\varepsilon(\bar{p})c}{\varepsilon(\bar{p}) - 1 - \tau\zeta(p)} \leqslant \frac{\varepsilon(p)c}{(\varepsilon(p)-1)(1-\tau)} = p$$

which, using the shape of the surplus function, is equivalent to:

$$\varepsilon(p) - \varepsilon(\bar{p}) \leqslant \tau \left[(\varepsilon(\bar{p}) - \varepsilon(p))(\varepsilon(p) - 1) - \varepsilon(p)\zeta'(p)p/\zeta(p) \right]$$

Given any positive commission, it is easy to verify that this condition is always satisfied as an inequality when $\varepsilon'(p), \zeta'(p) < 0$, is satisfied as an equality when $\varepsilon'(p) = \zeta'(p) = 0$, and is never satisfied when $\varepsilon'(p), \zeta'(p) > 0$. Indeed, in case of a power surplus function with a constand demand elasticity marketplace and seller set always the same markup, in case of exponential and translated power functions with increasing elasticities the marketplace sets a higher markup and the opposite outcome can emerge in other cases.[17] We summarize the essential findings as follows:

Proposition 2. *Under monopolistic competition with free entry of sellers on a marketplace serving customers with quasilinear IA preferences, a product with given cost and surplus function is sold at the same price for any commission rate by marketplace and sellers when they face the same constant demand elasticity, otherwise either the marketplace or the sellers can set a higher price.*

For later applications it is also useful to consider asymmetric situations where the marketplace faces different demand and cost functions than the sellers. Let us consider marketplace's products with a surplus function $v_j(p) = z\bar{v}(p)$ and marginal cost $\bar{c}$, where the demand shift parameter $z > 0$ measures the intensity of demand for the product of the marketplace, and is neutral on pricing. A power surplus function:

$$\bar{v}(p) = p^{1-\bar{\varepsilon}} \tag{18}$$

where $\bar{\varepsilon} > 1$ represents the constant demand elasticity faced by the marketplace, provides a price $\bar{p}(\tau) = \frac{\bar{\varepsilon}\bar{c}}{(\bar{\varepsilon}-1)(1-\tau)}$. If sellers face a power function (6), the markup is the same in case of a common elasticity ($\varepsilon = \bar{\varepsilon}$), otherwise the platform sets higher markups when facing a more rigid demand and lower markups when facing a more elastic demand than the sellers. Consider now a Logit framework with the following surplus function for the marketplace:

$$\bar{v}(p) = e^{-p/\bar{\mu}} \tag{19}$$

[17] For instance the surplus function $v(p) = (p+h)^{1-\varepsilon}$ implies $\varepsilon'(p), \zeta'(p) \lesseqgtr 0$ and $\bar{p}(\tau) \lesseqgtr p(\tau)$ if $h \lesseqgtr 0$.

where the parameter $\bar{\mu} > 0$ refers to the marketplace's products. The prices of the marketplace can be computed as $\bar{p}(\tau) = \bar{\mu} + \bar{c} + \tau p(\tau)$, where the marginal cost is augmented by the opportunity cost of giving up to commission revenues, as in AB (2021). Finally, translated power surplus functions for the marketplace as:

$$\bar{v}(p) = \frac{(a-p)^{1+\bar{\gamma}}}{1+\bar{\gamma}} \tag{20}$$

where $\bar{\gamma} > 0$ provide the price $\bar{p}(\tau) = \frac{a+\bar{\gamma}\bar{c}+a\tau\zeta}{1+\bar{\gamma}+\tau\zeta}$.

4.3 Commission by the marketplace

Given equilibrium pricing, we focus now on the profits of the marketplace at the time of selecting the commission. First of all, we can now rewrite the index of differential profits (16) as a function of the commission only:

$$I(\tau) = \sum_{j=1}^{m} v_j(\bar{p}_j(\tau)) \left[\frac{\zeta_j(\bar{p}_j(\tau)) \left[1 + \tau\zeta(p(\tau))\right]}{\varepsilon_j(\bar{p}_j(\tau))} - \tau\zeta(p(\tau)) \right] \tag{21}$$

where we defined the surplus elasticity for marketplace's products $\zeta_j(p) \equiv -\frac{v_j'(p)p}{v_j(p)}$, and we focus on cases where (21) is positive (otherwise it would be better to remain a pure marketplace). It is also useful to define the additional surplus from the marketplace's products as $\Psi(\tau) \equiv \sum_{j=1}^{m} v_j(\bar{p}_j(\tau))$. Notice that, by the envelop theorem, the impact of the commission on the index of differential profits can be computed from (16) as:

$$\begin{aligned} I'(\tau) &= -\Psi(\tau) \left[\zeta(p(\tau)) + \tau\zeta'(p(\tau))p'(\tau)\right] = \\ &= -\Psi(\tau)\zeta(p(\tau)) \left[1 + \frac{\tau}{1-\tau}\eta(p(\tau)) \left[1 + \zeta(p(\tau)) - \varepsilon(p(\tau))\right]\right] \end{aligned} \tag{22}$$

where we used the slope of the surplus elasticity and the pass-through rate. We focus on cases where this expression is negative (as it is always for a low enough commission): since the products's prices are set to maximize the index of differential profits, the only impact of the commission is the direct impact on the lost revenues.

The analysis simplifies further when we adopt a common surplus function for all the marketplace's products $z\bar{v}(p)$ and marginal cost $\bar{c}$, which still allows for differences from the sellers: we denote this as the case of symmetric products of the marketplace: since all these products are now sold at the same price, we simplify $\Psi \equiv mz\bar{v}(\bar{p})$ and $I = \Psi[\bar{\zeta}(\bar{p}) \left[1 + \tau\zeta(p)\right] / \bar{\varepsilon}(\bar{p}) - \tau\zeta(p)]$, where upperbars identify the elasticities of the marketplace.

Finally, given the index (21) we express the profits of the marketplace as:

$$\Pi(\tau) = \left[\tau\zeta(p(\tau)) \left(D(\tau) - H\right) + I(\tau)\right] G'(D(\tau)) \tag{23}$$

which is a function of the commission rate only, and will be assumed concave in what follows. We now move to the study of the commission set by the

marketplace with the purpose of comparing choices made by pure and hybrid marketplaces. Our previous analysis has shown that the commission rates are sufficient statistics for consumer welfare, therefore this comparison allows us to answer the question whether a hybrid platform harms consumers by setting worse conditions for the sellers or not. To build intuitions, we start by considering the logarithmic preferences of Nocke and Schutz (2018) based on (4) and then we move to the general case.

4.3.1 Loglinear preferences

Under loglinear preferences (4), consumer welfare can be expressed as $V = \log D(\tau) + E$, where the equilibrium value of the aggregator can be computed from (12) as:

$$D(\tau) = \frac{[(1-\tau)p(\tau) - c]\,|v'(p(\tau))|}{f} \tag{24}$$

and is decreasing and convex in the commission on sellers. The profits of the marketplace (23) are simplified as follows:

$$\Pi(\tau) = \tau\zeta(p(\tau))\left(1 - \frac{H}{D(\tau)}\right) + \frac{I(\tau)}{D(\tau)} \tag{25}$$

A pure marketplace facing $I(\tau) = 0$ sets the profit-maximizing commission rate τ^p that satisfies the first order condition:

$$\zeta(p(\tau^p)) + \tau^p\zeta'(p(\tau^p))p'(\tau^p) = \frac{|D'(\tau^p)|\,H\tau^p\zeta(p(\tau^p))}{D(\tau^p)\,[D(\tau^p) - H]} \tag{26}$$

assuming that both sides are positive and the second order condition for the interior maximum is satisfied. The left hand side of (26) represents the marginal revenue from the commission on an active seller and the right hand side the marginal costs of reducing the value of the price aggregator and therefore reducing the number of sellers active on the platform with the associated commission revenues.

Using the shape of the surplus elasticity and the pass-through as already in (22) and the impact of the commission on the price aggregator (24), we can also rearrange the implicit expression for the commission rate as follows:

$$\tau^p = \frac{1}{1 - \eta(p(\tau^p))[1 + \zeta(p(\tau^p)) - \varepsilon(p(\tau^p))] + \varepsilon(p(\tau^p))\frac{H}{D(\tau^p) - H}} \tag{27}$$

which depends on the various elasticities (all evaluated at the same commission rate) and on the relevance of the exogenous surplus obtained from other purchases H compared to the surplus obtained from the marketplace represented by the equilibrium aggregator $D(\tau^p)$. In particular, the commission decreases when the sellers face a more elastic demand because the platform internalizes the negative impact on their sales, and it decreases also when the buyers expect

a higher surplus from goods purchased outside the platform compared to what they obtain on the platform.[18]

In the example with power surplus functions (6), the profits are:

$$\Pi(\tau) = \tau(\varepsilon - 1)\left[1 - \frac{H}{D(\tau)}\right]$$

Using $D'(\tau) = \frac{-\varepsilon D(\tau)}{1-\tau}$, the first order condition for profit maximization is:

$$\Pi'(\tau) = (\varepsilon - 1)\left[1 - \frac{H}{D(\tau)} - \frac{\tau\varepsilon H}{(1-\tau)D(\tau)}\right] = 0$$

and the second order condition:

$$\Pi''(\tau) = (\varepsilon - 1)H\left[\frac{D'(\tau)}{D(\tau)^2} + \frac{\tau\varepsilon D'(\tau)}{(1-\tau)D(\tau)^2} - \frac{\varepsilon}{(1-\tau)^2 D(\tau)}\right] < 0$$

is always satisfied since each term in the square parenthesis is negative. This provides the optimal commission rate:

$$\tau^p = \frac{D(\tau^p) - H}{D(\tau^p) - H + \varepsilon H} \tag{28}$$

which can also be derived from the general rule (27). Since the right hand side of (28) is a monotonic decreasing function of the commission rate in the unit interval under our assumptions, there must be always a unique interior solution for $\tau^p \in (0, 1)$.

With the exponential specification (7) an implicit expression for the equilibrium commission rate can be obtained through the pass-through elasticity and demand elasticity derived above (for a proof that the first order conditions uniquely characterize the optimum in this case, see AB, 2021). And an analogous expression can be obtained for the translatd power specification (8).

Let us now move to a hybrid marketplace with $I(\tau) > 0$. This sets its commission rate to maximize (25) taking into account not only the effects on the price aggregator, but also the opportunity cost of losing commission revenues by setting a higher rate. Using (22), the profit-maximizing commission τ^h satisfies a first order condition that can be rearranged as follows:

$$\zeta(p(\tau^h)) + \tau^h \zeta'(p(\tau^h))p'(\tau^h) = \frac{\left|D'(\tau^h)\right|\left[H\tau^h\zeta(p(\tau^h)) - I(\tau^h)\right]}{D(\tau^h)\left[D(\tau^h) - \Psi(\tau^h) - H\right]} \tag{29}$$

again assuming an interior solution.

The comparison between commissions set by a pure marketplace in (26) and a hybrid marketplace in (29) is ambiguous in general. Heuristically, the introduction of own products generates an incentive to shift demand toward them through a higher commission: this is the effect at the numerator of (29)

[18] In case of zero marginal costs, which is relevant for sales of software apps (for instance games on app stores), we simply have $\tau^p = 1 - H/D(\tau^p)$.

depending on the differential profit index $I(\tau^h)$. However, the introduction of own products also reduces third party sales, decreasing the infra-marginal revenue loss when the commission is lowered in order to expand entry: this is the effect at the denominator of (29) depending on the incremental surplus generated by own products $\Psi(\tau^h)$. The first effect is the demand substitution effect aimed at diverting demand where it is more profitable for the marketplace, and the second effect is an extensive margin effect aimed at expanding demand for all products.

We now focus on the case of symmetric products by the marketplace. Repeating the steps above, we can rearrange the formula for the commission as:

$$\tau^h = \frac{D - H - \Psi\left[1 - \frac{\varepsilon(p)\bar{\zeta}(\bar{p})}{\zeta(p)\bar{\varepsilon}(\bar{p})}\right]}{\{1 - \eta(p)[1 + \zeta(p) - \varepsilon(p)]\}(D - H - \Psi) + \varepsilon(p)\left[H + \Psi\left(1 - \frac{\bar{\zeta}(\bar{p})}{\bar{\varepsilon}(\bar{p})}\right)\right]} \tag{30}$$

where upperbars refer to the products of the marketplace and we dropped the arguments related to the commission on the right hand side. We still cannot determine whether a positive or increasing value of the surplus from marketplace's products Ψ increases or reduces the commission. Nevertheless we can show that already within our examples both cases can emerge.

Under common power functions (6) for sellers with demand elasticity ε and (18) for the marketplace with demand elasticity $\bar{\varepsilon}$, the last formula boils down to:

$$\tau^h = \frac{D - H - \Psi\frac{\varepsilon - \bar{\varepsilon}}{(\varepsilon - 1)\bar{\varepsilon}}}{D - H + \varepsilon H + \Psi\frac{\varepsilon - \bar{\varepsilon}}{\bar{\varepsilon}}} \lessgtr \tau^p \quad \text{if} \quad \varepsilon \gtrless \bar{\varepsilon} \tag{31}$$

where the second order condition remains satisfied for small differences in elasticities.

In the specification where not only the products of the sellers are symmetric but also the products of the marketplace provide power surplus functions with the same elasticity (namely $\varepsilon = \bar{\varepsilon}$), we have a constant commission given by (28): a hybrid marketplace sets the same markups as the sellers and does not change its commission compared to a pure marketplace. The hybrid marketplace is therefore completely neutral on consumer welfare, in spite of potential differences in both costs and demand (scale) parameters between the products of sellers and those of the marketplace.

Of course, if the marketplace faces a different demand elasticity than the sellers, the commission can change. In particular, when the marketplace faces a less elastic demand than the sellers for its products ($\bar{\varepsilon} < \varepsilon$), it reduces the commissions while introducing the products, and when it faces a more elastic demand ($\bar{\varepsilon} > \varepsilon$) it increases the commissions. Intuitively, when third party sellers face a relatively more elastic demand than the marketplace, the latter sets higher markups on its products, but recovers entry of sellers and expenditure of buyers on the marketplace by reducing the commission. This may well be the case for Amazon if customers have indeed a more rigid demand for its products compared to those of sellers hosted on the platform, for instance due to Amazon reputation for more reliable shipping and post-sale services.

One can verify that in case of exponential surplus functions, a hybrid marketplace sets always a higher commission than a pure one, as already shown by AB (2021).[19] Instead, in case of translated power functions a hybrid marketplace sets a higher commission when $\bar{\gamma} = \gamma$ but not necessarily when $\bar{\gamma} > \gamma$. The broad message is that the introduction of products by a marketplace increases its profits also through adjustments of the commissions, but exerts an ambiguous impact on consumer welfare, whose quantitative measure is unlikely to be relevant in practice.

4.3.2 General preferences

In case of an increasing and concave transformation $G(D)$, the analysis is slightly more cumbersome. The equilibrium aggregator is defined by (12) for a given commission, and the impact of the commission on the aggregator and entry in (13) depends on the index of curvature of the transformation $\sigma(D)$. The expression for the profits of a hybrid marketplace is given by (23). Assuming an interior solution, the formula for the profit-maximizing commission rate can be expressed as follows:

$$\zeta(p(\tau))+\tau\zeta'(p(\tau))p'(\tau) = \frac{|D'(\tau)|\left\{[1-\sigma(D(\tau))+\frac{\sigma(D(\tau))}{D(\tau)/H}]\tau\zeta(p(\tau))-\frac{\sigma(D(\tau))}{D(\tau)/I(\tau)}\right\}}{D(\tau)-\Psi(\tau)-H}$$

where we keep assuming the existence of an interior solution. The left hand side is always the marginal revenue from the commission on a seller and the right hand side is the marginal cost of reducing entry of sellers and welfare, affected in opposite directions by the differential profits $I(\tau)$ and the incremental surplus on own products $\Psi(\tau)$.

Under a power surplus function for sellers with demand elasticity ε the commission selected by a pure marketplace can be derived through usual computations as:

$$\tau^p = \frac{1}{1+\varepsilon\left[\frac{D}{\sigma(D)(D-H)}-1\right]}$$

where we dropped arguments. This corresponds to (28) when $\sigma(D) = 1$ under loglinear preferences, but is reduced for lower values of $\sigma(D)$. Intuitively, when entry of sellers is more sensitive to the commission, it is optimal to set a lower commission rate.

If the marketplace introduces products facing the constant demand elasticity

[19] In particular, since in the Logit framework we have $\zeta(p)/\varepsilon(p) = \bar{\zeta}(\bar{p})/\bar{\varepsilon}(\bar{p}) = 1$ for any prices, the equilibrium commission satisfies:

$$\tau^h = \frac{D-H}{[1-\eta(p)](D-H-\Psi)+\varepsilon(p)H}$$

which is necessarily higher when Ψ is positive compared to when it is null. For a discussion of the second order conditions for this case see AB (2021).

$\bar{\varepsilon}$, we obtain:

$$\tau^h = \frac{1 - \frac{\Psi}{D-H}\frac{\varepsilon-\bar{\varepsilon}}{(\varepsilon-1)\bar{\varepsilon}}}{1 + \varepsilon\left[\frac{D}{\sigma(D)(D-H)} - 1\right] + \frac{\Psi}{D-H}\frac{\varepsilon-\bar{\varepsilon}}{\bar{\varepsilon}}} \lesseqgtr \tau^p \quad \text{if} \quad \varepsilon \gtreqless \bar{\varepsilon} \tag{32}$$

which confirms the neutrality under a common elasticity and the incentive to reduce the commission for a marketplace if and only if this faces less elastic demands. We summarize our final findings as follows:

Proposition 3. *Under monopolistic competition with free entry of sellers on a marketplace serving customers with quasilinear IA preferences, the introduction of symmetric products by the marketplace is neutral on consumer welfare if the sellers face the same constant demand elasticity as the marketplace, and otherwise can either increase or decrease consumer welfare.*

Our benchmark analysis assumed an *ad valorem* commission on the sellers. In the Appendix we consider the case where the only available tool is a commission on the quantity sold rather than the revenues, as in Zennyo (2021). Also in that case the welfare impact of the introduction of products by the platform depends on the change of the commission and is in general ambiguous. However, the impact is neutral in case of a Logit microfoundation, which is consistent with the result of Zennyo (2021), as well as with the equivalence of unit taxes under Logit demand and ad valorem taxes under CES demands documented by Anderson and de Palma (2015).

Our general policy implication differs from the one emphasized by AB (2021), because banning the dual mode to convert a hybrid marketplace into a pure marketplace may actually harm rather than benefit consumers. However, other results obtained by AB (2021) extend naturally to our framework. In particular, a hybrid marketplace has an interest in promoting higher perceived quality or lower (production and shipping) costs for both its own products and those of third party sellers. Moreover, the introduction of a tax on third party revenues tends to increase the commission set by the marketplace, while a tax on the revenues of products directly sold by the marketplace tends to reduce the commission, with opposite effects on consumer welfare.

We conclude this section by emphasizing the application to an upstream monopolist that provides an input to differentiated downstream producers with endogeneous entry of these producers. Our results suggest that a vertical merger of the upstream monopolist with some of the downstream producers is neutral on consumer welfare when the price of the input is fixed, because entry keeps the aggregator constant. However, when the price of the input for the downstream producers is chosen by the monopolist, the merger may either increase or decrease it, with opposite implications on consumer welfare. If a raise rivals's cost effect is dominant the merger is harmful for consumers, while if the extensive margin effect that is dominant the merger is beneficial.

5 Extensions

In this section we extend the model in a few directions. First, we introduce strategic price competition between sellers, which is relevant when few of them are active in the same product category. Next, we change the timing of the baseline model by considering a marketplace that cannot commit to price choices before entry of the sellers. Subsequently, we study the choice of product selection by the marketplace, essentially endogenizing which products are actually introduced. Finally, we consider advertising as an additional source of monetization which is becoming always more important for online marketplaces.

Another extension of some interest is the one to competing subscription-funded marketplaces, which is the direction taken by Amazon with its Prime membership fee (or by videogame platforms such as Game Pass or device-funded platforms as the one of Apple). The demand-substitution and extensive margin effects are present also in that context, but the platforms internalize also the direct impact of their strategies on consumer welfare, because this affects monetization through the access fees, and competition leads the platforms to shift revenues to consumers through lower access fees, which amplifies benefits for consumers.

5.1 Strategic interactions

Our main framework assumed monopolistic competition between sellers, which appears the relevant scenario for marketplaces which host a huge number of products. However, when product categories subject to the same commission are narrowly defined, or platforms introduce products in direct competition with third party rivals and can change their access conditions, sellers may take strategic interactions into account at the pricing stage. Here we verify how Bertrand competition between sellers affects our results.

At the pricing stage, each seller maximizes (9) taking into account the impact of its price choice on the true demand function, and therefore also the aggregator. Exploiting symmetry across sellers, this delivers the price rule $p(\tau)$ such that:

$$p = \frac{\epsilon(p,D)c}{(1-\tau)(\epsilon(p,D)-1)} \quad \text{with } \epsilon(p,D) \equiv \varepsilon(p) - \zeta(p)v(p)\frac{\sigma(D)}{D} \tag{33}$$

where the reduced demand elasticity implies a higher markup: as usual, strategic sellers set higher prices compared to monopolistically competitive sellers. Free entry implies always the zero profit condition (12), so the system of two equations determines jointly (p, D) as functions of the commission τ.[20] This preserves the neutrality of aggregator and welfare with respect to the provision of products by the marketplace, which should not be surprising since this neutrality in free entry models was originally observed in the presence of strategic interactions (Etro, 2008, 2011).

[20] For instance, under loglinear preferences and power surplus functions one can derive $p = \frac{\varepsilon c}{(\varepsilon-1)(1-\tau)-f}$ for a small enough fixed cost per customer.

Once we know how the commission affects the prices of sellers $p(\tau)$ and the aggregator $D(\tau)$, nothing changes qualitatively in the derivation of prices of the marketplace (which tends to be more aggressive in pricing compared to the sellers) and of its commission. In practice, competition is softened between a small number of sellers increasing also the prices of the marketplace and reducing welfare compared to the case of monopolistic competition, but the ambiguous impact of hybrid platforms on the commission remains.

5.2 No price commitments

Our baseline analysis has analyzed a marketplace acting as a Stackelberg leader able to pre-commit on the commission on third party sellers and also on the prices of its own products. In practice, it is not clear that a marketplace as Amazon has any first mover advantage in setting prices before the entry choice of the sellers: most of the price changes occur in real time time for both Amazon and the sellers hosted on its platform, while entry choices are long run decisions. A pre-commitment allows the marketplace to increase its own prices in function of the pre-determined commission rate and to better monetize the demand of its own products, but it is not crucial to obtain the result of lower commissions and benefit consumers compared to a pure marketplace. To verify this, we now change the timing of the baseline model, assuming that both the marketplace and the sellers set their prices in the last stage after the enty decisions.

While sellers set prices according to the usual rule $p(\tau)$ in (10), the marketplace does not internalize the impact on entry and sets lower prices $\bar{p}_j = \bar{p}_j(0)$ that satisfy:

$$\bar{p}_j = \frac{\varepsilon_j(\bar{p}_j)\bar{c}_j}{\varepsilon_j(\bar{p}_j) - 1} \tag{34}$$

because it avoids double marginalization on its own products. Free entry, however, determines the same aggregator $D(\tau)$ as before, which is determined by (12). The expressions for the equilibrium commissions are the same as in the baseline model, with the only difference that the index of differential profits takes into account the new prices set by the marketplace as in:

$$I(\tau) = \sum_{j=1}^{m} v_j(\bar{p}_j(\tau)) \left[\frac{\zeta_j(\bar{p}_j)}{\varepsilon_j(\bar{p}_j)} - \tau\zeta(p(\tau)) \right] \tag{35}$$

This expression is smaller than (21) for a given commission because the marketplace cannot precommit to higher prices exploiting the demand diversion generated by the commission. Which pushes for lower commissions set by a hybrid marketplace.

To illustrate, let us consider the case of loglinear preferences with symmetric products on the marketplace. Replacing (35) in (29) we can rearrange the commission for the hybrid marketplace as:

$$\tau^h = \frac{D - H - \Psi \left[1 - \frac{\varepsilon(p)\bar{\zeta}(\bar{p})}{\zeta(p)\bar{\varepsilon}(\bar{p})} \right]}{\{1 - \eta(p)[1 + \zeta(p) - \varepsilon(p)]\}(D - H - \Psi) + \varepsilon(p)\,(H + \Psi)} \tag{36}$$

Under a common and constant demand elasticity for all products or under Logit demand systems the parenthesis at the numerator is null and the terms at the denominator push alone for a lower commission rate compared to a pure marketplace. Intuitively, when the marketplace cannot commit to optimally monetize the demand diversion generated by the commissions through appropriate price commitments for its products, the demand substitution mechanism tends to be dominated by the extensive margin mechanism.

5.3 Product selection

Our next investigation is about the conditions under which the marketplace enters and with which products in the baseline model. The problem was explored by Hagiu and Wright (2015) and Etro (2021a) for a given set of product varieties and sellers under the assumptions of independent demands and (specific) commissions optimally set on each product. In our framework with free entry of sellers, interdependent demands and a uniform (percentage) commission set on all products, the issue is complicated because the marketplace must take in consideration not only the relative profitability of direct and third party sales, but also the impact on demand allocation across products.[21]

Given the gross profits of the marketplace (15), let us consider for simplicity the case where the marketplace bears the same fixed costs as the sellers for each product. Then the introduction of a new product is profitable if it augments the net profits:

$$\Pi(\tau^h) - mf = [\tau^h \zeta(p(\tau^h)) \left(D(\tau^h) - H\right) + I(\tau^h)] G'(D(\tau^h)) - mf \qquad (37)$$

by increasing enough the index of differential profits to cover the fixed cost. Taking as given the commission (which is marginally affected by the introduction of a single product), using the index (21) and omitting arguments, the condition for the introduction of a product with surplus function $z\bar{v}(\bar{p})$ to be profitable is:

$$z\bar{v}(\bar{p}) \left[\frac{\bar{\zeta}(\bar{p}) \left[1 + \tau^h \zeta(p)\right]}{\bar{\varepsilon}(\bar{p})} - \tau^h \zeta(p) \right] - \frac{f}{G'(D)} > 0$$

Employing the zero profit condition for the sellers (12) to replace the fixed cost and rearranging, we can obtain the exact condition under which the platform increases profits by providing the good:

$$\left[\frac{z\bar{v}(\bar{p})\bar{\zeta}(\bar{p})}{\bar{\varepsilon}(\bar{p})} - \frac{v(p)\zeta(p)}{\varepsilon(p)} \right] + \tau^h \zeta(p) \left[\frac{v(p)}{\varepsilon(p)} - z\bar{v}(\bar{p}) \frac{\bar{\varepsilon}(\bar{p}) - \bar{\zeta}(\bar{p})}{\bar{\varepsilon}(\bar{p})} \right] > 0 \qquad (38)$$

The first term represents the difference in gross profits between marketplace and sellers, and under a zero commission this must be positive for entry by the marketplace to cover the fixed cost and create positive net profits. The

[21] The related problem of product selection in a market with monopolistic competition is addressed in Spence (1976) and more recently in Bertoletti and Etro (2022).

second term can be either positive or negative, and accounts for the role of the commission: on one side, a positive commission reduces the incentives of the marketplace to enter because profitability must compensate for the lost commission revenues, but, on the other side, it increases the incentives because it shifts demand toward the products of the marketplace whose markup can be increased.

It is immediate to verify that in our example with power functions (6) for sellers with demand elasticity ε and (18) for the marketplace with demand elasticity $\bar{\varepsilon}$, the condition becomes:

$$z\bar{p}^{1-\bar{\varepsilon}}\left[1-\frac{1+\tau^h(\varepsilon-1)}{\bar{\varepsilon}}\right] > p^{1-\varepsilon}\left[1-\frac{1+\tau^h(\varepsilon-1)}{\varepsilon}\right]$$

If marketplace and sellers face the same surplus functions and costs, the expressions on each side are identical and the marketplace is indifferent between introducing or not its product. Otherwise the condition is satisfied if the marketplace has a large enough advantage either in demand or costs. In any case, when the marketplace finds it profitable to introduce the product, this does not affect consumer welfare (because it does not change the commission).

Similarly, the Logit example provides the simplified condition $z\exp(p/\mu - \bar{p}/\bar{\mu}) > 1-\tau^h$, which is satisfied only if the marketplace has large enough advantages either in demand or costs compared to the sellers. The consequence is that even in these cases where the introduction of products by the marketplace would reduce consumer welfare (by raising the commission), the products may not be introduced to start with.

5.4 Advertising

An expanding source of monetization for online marketplaces is represented by advertising by sellers, essentially aimed at product discovery. While platforms have their own incentives to promote third party sales that generate commission revenues, each seller has an additional incentive to spend in ads to divert "clicks" of customers to its own products from those of the rivals. To the extent that this expands total sales, it can also generate additional revenues for the marketplace. This creates an imperfect substitutability between commission revenues and ad revenues for the platform and can also affect its incentives to change conditions for third party sellers. In the working paper version of this article we have shown that this is not necessarily the case and the structure of monetization can be independent from the introduction of marketplace's products.[22]

More formally, we can augment the model with a probability of purchase for each product depending on spending in ads by the seller. The ad fee can be regarded as exogenous if the willingness to pay of the sellers depends on

[22] Notice that ads can also raise other issues for platforms. Kirpalani and Philippon (2020) argue that information disclosure by consumers improves the gains from match quality but may increase too much the market power of a monopolistic marketplace toward third party sellers. Latham *et al.* (2021) explore the role of Google in the ad tech stack.

returns on alternative ad campains and is not biased by the platform, but we can also consider the case in which the marketplace exploits its market power and selects the ad fee. In the case of Amazon, ad fees are determined through ad auctions per click, and a concern is that Amazon may exploit its dual role by manipulating quality scores to increase ad costs for rival sellers.

Given percentage commissions and ad fees, each seller selects price and ads under monopolistic competition. As in the baseline framework, the free entry condition determines the price aggregator, and therefore welfare, as a decreasing function of each fee and, once again, independently from the products introduced by the marketplace. When the marketplace sets its commission rate, the monetization through ads tends to reduce the marginal revenues on commission and therefore the optimal commission rate: intuitively the marketplace is aware of the impact of higher ad costs on prices and therefore sales. However, the introduction of products by the marketplace exerts the usual ambiguous impact on the commission.

More generally, commissions and ad fees can be selected by the marketplace according to optimal taxation principles. But these should be independent from the introduction of own products, implying that pure and hybrid marketplace should decide on the structure of monetization on third party sellers independently from the source of its revenues. Once again, the usual trade-off between demand-shifting and extensive margin mechanisms determines whether hybrid platforms are going to increase or reduce consumer welfare through changes of the total payment of sellers. This confirms the spirit of the results of our benchmark model also when the marketplace monetizes through ads for product discovery.

6 Conclusion

We have analyzed the role of endogenous entry of monopolistically competitive sellers on hybrid marketplaces under rather general demand systems. Our microfoundation was based on indirectly additive aggregators and allowed us to show that a hybrid marketplace can set either higher or lower commissions compared to a pure marketplace, with opposite effects on consumer (and user) welfare. For instance, under constant demand elasticities, a hybrid marketplace sets lower percentage commissions increasing consumer welfare if and only if its products face a less elastic demand.

The literature so far has advanced various arguments for which a hybrid marketplace may benefit consumers by fostering competition on the platform (Hagiu *et al.*, 2021) and introducing cheaper or more valuable products (Etro, 2021a; Shopova, 2021), but may also harm consumers favoring its own products or undermining entry by sellers (AB, 2021). Considering differentiation between products and free entry of third party sellers on a marketplace, we have suggested that a key channel through which the hybrid marketplace can affect welfare is the change of the commissions set on rival sales. While the direction of this change remains an empirical issue, the commission rates set by Amazon

per product categories have been quite stable over time and not significantly correlated with the introduction of private label products by Amazon.

Further research may explore other strategies by marketplaces as those concerning search services for consumers, investments in logistics and platform-liability design: see Hervas-Drane and Shelegia (2022) and Zennyo (2021) for early explorations in these directions. It should be remarked that our micro-foundation and the assumption of symmetric sellers have generated aggregative games where the introduction of own products by the marketplace is neutral on consumer welfare for a given commission, and any welfare benefits emerge indirectly through reductions of the commissions. One could also explore more general frameworks where a hybrid marketplace can benefit consumers directly, that is by introducing new products at lower prices and by strengthening competition on the platform (see Shopova, 2021 and Hagiu *et al.*, 2021) or explore asymmetries between sellers. Finally, there is space for fruitful empirical work on the welfare impact of the strategies of Amazon. Lee and Musolff (2021) and Gutierrez (2021) have analyzed empirically the trade-offs generated by vertical integration of Amazon in nested Logit frameworks, respectively with endogenous entry of sellers for given commissions and with endogenous commissions for given sellers. It would be important to account for endogenous entry, prices and commissions under more general demand conditions.

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Appendix: Specific commissions

In this Appendix we consider the case where the only commission available is a specific commission t on the quantity sold rather than the revenues, as in Zennyo (2021), who also employes Bertrand competition instead of monopolistic competition. We keep the rest of the notation as in the baseline model and follow its development.

Given the specific commission on sales, each seller i sets the price to maximize gross profits:

$$\pi(p_i) = (p_i - t - c)\left|v'\left(p_i\right)\right| G'(D(\mathbf{p}))$$

ignoring the impact on the price aggregator. This provides price rules $p = p(t)$ that satisfy:

$$p = \frac{\varepsilon(p)(c+t)}{\varepsilon(p)-1}$$

Free entry of sellers implies the zero profit condition:

$$(p(t) - t - c)\left|v'\left(p(t)\right)\right| G'(D(t)) = f$$

which determines the equilibrium aggregator $D(t)$ as a function decreasing in the specific commission and, as before, independent from the provision of marketplace's products. The profit of the hybrid platform can be expressed as:

$$\Pi = t\xi(p(t))\left(D(t) - H\right) G'(D(t)) + I(\mathbf{p}, t)G'(D(t))$$

where we defined $\xi(p) \equiv -v'(p)/v(p)$ and the relevant index of differential profits:

$$I(\mathbf{p}, t) \equiv \sum_{j=1}^{m}\left[\left(\bar{p}_j - \bar{c}_j\right)\left|v_j'(\bar{p}_j)\right| - t\xi(p(t))v_j(\bar{p}_j)\right]$$

The prices of the platform's products are selected to maximize this index according to the rule:

$$\bar{p}_j(t) = \bar{c}_j + \frac{1 + t\xi(p(t))}{\vartheta_j(\bar{p}_j(t))}$$

where we defined $\vartheta_j(p) \equiv -v_j''(p)/v_j'(p)$. This allows us to express the index of differential profits as a function of the specific commission only:

$$I(t) = \sum_{j=1}^{m} v_j(\bar{p}_j(t))\left[\frac{\xi_j(\bar{p}_j(t))\left[1 + t\xi(p(t))\right]}{\vartheta_j(\bar{p}_j(t))} - t\xi(p(t))\right]$$

where we defined $\xi_j(p) \equiv -v_j'(p)/v_j(p)$. Accordingly, the profits can be written as:

$$\Pi(t) = \left[t\xi(p(t))\left(D(t) - H\right) + I(t)\right]G'(D(t))$$

whose maximization with respect to t defines the equilibrium with implications that are qualitatively analogous to those obtained under percentage commissions.

To obtain further results it is convenient to focus on the case of loglinear preferences (4), where the aggregator can be derived as:

$$D(t) = \frac{(p(t) - t - c)\,|v'(p(t))|}{f}$$

In such a case, a pure marketplace sets a commission to maximize $t\xi(p(t))(1 - H/D(t))$, implying the condition:

$$\xi(p(t^p)) + t^p \xi'(p(t^p))p'(t^p) = \frac{|D'(t^p)|\, H t^p \xi(p(t^p))}{D(t^p)\,[D(t^p) - H]}$$

which equalizes as usual the marginal revenue from the specific commission per seller on the left hand side and the marginal cost of reducing the the number of sellers on the right hand side. A hybrid marketplace, instead, maximizes $t\xi(p(t))(1 - H/D(t)) + I(t)/D(t)$ setting the commission according to the following rule:

$$\xi(p(t^h)) + t^h \xi'(p(t^h))p'(t^h) = \frac{\left|D'(t^h)\right| \left[H t^h \xi(p(t^h)) - I(t^h)\right]}{D(t^h)\,[D(t^h) - \Psi(t^h) - H]}$$

where we defined the additional surplus from the products of the marketplace as $\Psi(t) \equiv \sum_{j=1}^{m} v_j(\bar{p}_j(t))$. Once again, this is compatible with either a higher or a lower commission due to opposite effects that are analogous to what emerged under *ad valorem* commissions. For the same reasons the impact on consumer welfare can go in either direction.

To compare the results with those of Zennyo (2021), let us consider the Logit demand system where all the surplus functions are exponential, with (7) for the sellers and (19) for all the products of the marketplace. This implies prices of sellers and marketplace given by $p(t) = c + \mu + t$ and $\bar{p}_j(t) = \bar{c}_j + \bar{\mu} + \frac{\bar{\mu}}{\mu} t$, which would match for identical products as in Zennyo (2021). Moreover, computing $\xi(p) = |D'(t)|\,/D(t) = 1/\mu$ and $\vartheta_j(p) = \xi_j(p) = 1/\bar{\mu}$ and $I(t) = \Psi(t)$ we can simplify the implicit expression for the commission to:

$$t^* = \frac{\mu\,(D(t^*) - H)}{H}$$

The key aspect is that the commission is independent from the presence of marketplace's products, namely $t^* = t^h = t^p$. This confirms the result by Zennyo (2021) that under a Logit microfoundation a hybrid marketplace sets the same specific commission as a pure marketplace. Clearly, the neutrality does not hold with surplus functions that are not exponential and, *a fortiori*, in the case of a more general microfoundation. Accordingly the introduction of products by the marketplace may either increase or decrease consumer welfare.